A Possible Connection Between Baryons and Magnetic Monopoles

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I shall show as concisely as possible, the mathematical connection which leads me to suspect that baryons are intimately related to magnetic monopoles in non-Abelian field theory.

Start with a non-Abelian field strength tensor:

$$F_i^{\mu\nu} = \partial^\mu B_i^{\ \nu} - \partial^\nu B_i^{\ \mu} - g f_{ijk} B_j^{\ \mu} B_k^{\ \nu} \quad (1)$$

where the $B_i^{\ \mu}$ are the gauge bosons (classical potentials) of whatever Yang-Mills group one is using (for instance, weak SU(2) or SU(3) QCD), f_{ijk} are the group structure constants, and g is the group interaction charge.

Form a third-rank antisymmetric tensor $P_i^{\mu\nu\sigma}$ for this group according to:

$$P_i^{\mu\nu\sigma} = \partial^\sigma F_i^{\mu\nu} + \partial^\mu F_i^{\nu\sigma} + \partial^\nu F_i^{\sigma\mu} . \quad (2)$$

$P_i^{\mu\nu\sigma}$ is a tensor expressed in terms of second derivatives of the potentials and so is a "source" in the same sense as the current density four-vector defined by Maxwell's equation $J_i^{\ \nu} = \partial_\mu F_i^{\mu\nu}$ for "electric sources." When (1) is written for an Abelian field strength, $F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$, then (2) becomes Maxwell's magnetic equation $P^{\mu\nu\sigma} = \partial^\sigma F^{\mu\nu} + \partial^\mu F^{\nu\sigma} + \partial^\nu F^{\sigma\mu} = 0$ for "magnetic sources." This is all standard physics.

Now, substitute (1) directly into (2) and reduce out the Abelian terms in the usual manner. One finds that:

$$P_i^{\mu\nu\sigma} = -g f_{ijk} \left(\partial^\sigma B_j^{\ \mu} B_k^{\ \nu} + B_j^{\ \mu} \partial^\sigma B_k^{\ \nu} + \partial^\mu B_j^{\ \nu} B_k^{\ \sigma} + B_j^{\ \nu} \partial^\mu B_k^{\ \sigma} + \partial^\nu B_j^{\ \sigma} B_k^{\ \mu} + B_j^{\ \sigma} \partial^\nu B_k^{\ \mu} \right). \quad (3)$$

This non-Abelian magnetic source is NOT equal to zero. T'hooft & Polyakov and others have previously pointed out that non-Abelian field theory seems to give rise to magnetic monopoles, but to date, no connection has been made from this line of inquiry to anything which has been experimentally observed. So the question arises: might such a third-rank antisymmetric source represent anything we observe in the physical world?

First, let's first work on the terms $\partial^\sigma B_j^{\ \mu}$. One might start with a $B_j^{\ \mu} = \varepsilon^\mu \lambda_j e^{-iq^\tau x_\tau}$ where ε^μ is a polarization vector, λ_j are the group generator matrices, and q^τ is the vector boson four-momentum, and then take $\partial^\sigma B_j^{\ \mu}$ from this. However, an even more direct route is to use the quantum mechanical substitution:

$$-i\partial^\sigma B_j{}^\mu = \left[q^\sigma, B_j{}^\mu\right] = q^\sigma B_j{}^\mu - B_j{}^\mu q^\sigma \quad (4)$$

see, for example, Ryder's *Quantum Field Theory*, just after equation (2.166). Then, we substitute (4) into (3) to obtain:

$$P_i^{\mu\nu\sigma} = -gf_{ijk}\begin{pmatrix} \left(iq^\sigma B_j{}^\mu - iB_j{}^\mu q^\sigma\right)B_k{}^\nu + B_j{}^\mu\left(iq^\sigma B_k{}^\nu - iB_k{}^\nu q^\sigma\right) \\ +\left(iq^\mu B_j{}^\nu - iB_j{}^\nu q^\mu\right)B_k{}^\sigma + B_j{}^\nu\left(iq^\mu B_k{}^\sigma - iB_k{}^\sigma q^\mu\right) \\ +\left(iq^\nu B_j{}^\sigma - iB_j{}^\sigma q^\nu\right)B_k{}^\mu + B_j{}^\sigma\left(iq^\nu B_k{}^\mu - iB_k{}^\mu q^\nu\right) \end{pmatrix}.$$

Now, we distribute, reduce (all terms such as $B_j{}^\mu q^\sigma B_k{}^\nu$ with q^σ *between* the two vector bosons cancel identically), and express using commutators to obtain:

$$\begin{aligned} P_i^{\mu\nu\sigma} &= -igf_{ijk}\left(q^\sigma B_j{}^\mu B_k{}^\nu + q^\mu B_j{}^\nu B_k{}^\sigma + q^\nu B_j{}^\sigma B_k{}^\mu - B_j{}^\mu B_k{}^\nu q^\sigma - B_j{}^\nu B_k{}^\sigma q^\mu - B_j{}^\sigma B_k{}^\mu q^\nu\right) \\ &= igf_{ijk}\left(\left[B_j{}^\mu B_k{}^\nu, q^\sigma\right] + \left[B_j{}^\nu B_k{}^\sigma, q^\mu\right] + \left[B_j{}^\sigma B_k{}^\mu, q^\nu\right]\right) \end{aligned} \quad (5)$$

Now, let's work on the potentials $B_j{}^\mu$. We shall assume for sake of simplicity that the $B_j{}^\mu$ are *massless* bosons (such a gluons or photons), and leave it to the reader, for the moment, to consider the more general situation where these bosons are massive. Maxwell's electric equation $J^\mu = \partial_\tau\partial^\tau A^\mu$ with the gauge condition $\partial_\mu A^\mu = 0$ leads in a well-known way using $A^\mu = \varepsilon^\mu e^{-iq^\tau x_\tau}$ to $A^\mu = -\left(1/q^2\right)J^\mu$ where $q^2 = q^\tau q_\tau$. Here, to illustrate the general character of (5) we follow this same path with the non-Abelian index *i*, and thus use the equation:

$$B_j{}^\mu = -\frac{1}{q^2}J_j{}^\mu. \quad (6)$$

Substituting (6) into (5) yields:

$$P_i^{\mu\nu\sigma} = -igf_{ijk}\left(+\left[\frac{1}{q^2{}_{(\mu)}}J_j{}^\mu\frac{1}{q^2{}_{(\nu)}}J_k{}^\nu, q^\sigma\right] + \left[\frac{1}{q^2{}_{(\nu)}}J_j{}^\nu\frac{1}{q^2{}_{(\sigma)}}J_k{}^\sigma, q^\mu\right] + \left[\frac{1}{q^2{}_{(\sigma)}}J_j{}^\sigma\frac{1}{q^2{}_{(\mu)}}J_k{}^\mu, q^\nu\right]\right). \quad (7)$$

Here we have introduced the *label* ${}_{(\mu)}$ in $q^2{}_{(\mu)}$ to designate that $q^2{}_{(\mu)}$ is the squared boson momentum arising from the vector boson $B_j{}^\mu$ with the spacetime index μ, and similarly for other currents. We see in (7) that the magnetic source $P_i^{\mu\nu\sigma}$ now consists of currents labeled with exactly three distinct spacetime indexes, namely, $J_j{}^\mu$, $J_j{}^\nu$, $J_j{}^\sigma$, as well as exactly three vector boson momentum four vectors q^μ, q^ν, q^σ. A "three-constituent" source begins to take shape.

Next, for each current, we make a substitution of the well-known form:

$$J_j^{\ \mu} = \overline{\psi}\lambda_j\gamma^\mu\psi \quad (8)$$

where ψ is a fermion wavefunction. With (8), (7) now becomes:

$$P_i^{\mu\nu\sigma} = -igf_{ijk}\begin{pmatrix} +\left[\frac{1}{q^2_{(\mu)}}\left(\overline{\psi}\lambda_j\gamma^\mu\psi\right)\frac{1}{q^2_{(\nu)}}\left(\overline{\psi}\lambda_k\gamma^\nu\psi\right), q^\sigma\right] \\ +\left[\frac{1}{q^2_{(\nu)}}\left(\overline{\psi}\lambda_j\gamma^\nu\psi\right)\frac{1}{q^2_{(\sigma)}}\left(\overline{\psi}\lambda_k\gamma^\sigma\psi\right), q^\mu\right] \\ +\left[\frac{1}{q^2_{(\sigma)}}\left(\overline{\psi}\lambda_j\gamma^\sigma\psi\right)\frac{1}{q^2_{(\mu)}}\left(\overline{\psi}\lambda_k\gamma^\mu\psi\right), q^\nu\right] \end{pmatrix}. \quad (9)$$

Now, let's do some labeling so we can try to draw a Feynman diagram for (9). We label each fermion wavefunction in accordance with the spacetime index of the current with which it is associated. Thus, for example, $\overline{\psi}\lambda_j\gamma^\mu\psi$ becomes $\overline{\psi}_{(\mu)}\lambda_j\gamma^\mu\psi_{(\mu)}$. Also, giving attention to the non-abelian indexes *j*, *k*, every wavefunction ψ to the right of a λ_j generator receives a "1" label to indicate a fermion in "state 1," the adjoint $\overline{\psi}$ to the left of a λ_j receives a "state 2" label, every wavefunction ψ to the right of a λ_k generator receives the same "state 2" label and the adjoint $\overline{\psi}$ to the left of a λ_k receives a "state 3" label. With this labeling, (9) now becomes:

$$P_i^{\mu\nu\sigma} = -igf_{ijk}\begin{pmatrix} +\left[\frac{1}{q^2_{(\mu)}}\left(\overline{\psi}_{(\mu 2)}\lambda_j\gamma^\mu\psi_{(\mu 1)}\right)\frac{1}{q^2_{(\nu)}}\left(\overline{\psi}_{(\nu 3)}\lambda_k\gamma^\nu\psi_{(\nu 2)}\right), q^\sigma\right] \\ +\left[\frac{1}{q^2_{(\nu)}}\left(\overline{\psi}_{(\nu 2)}\lambda_j\gamma^\nu\psi_{(\nu 1)}\right)\frac{1}{q^2_{(\sigma)}}\left(\overline{\psi}_{(\sigma 3)}\lambda_k\gamma^\sigma\psi_{(\sigma 2)}\right), q^\mu\right] \\ +\left[\frac{1}{q^2_{(\sigma)}}\left(\overline{\psi}_{(\sigma 2)}\lambda_j\gamma^\sigma\psi_{(\sigma 1)}\right)\frac{1}{q^2_{(\mu)}}\left(\overline{\psi}_{(\mu 3)}\lambda_k\gamma^\mu\psi_{(\mu 2)}\right), q^\nu\right] \end{pmatrix} \quad (10)$$

This source now consists of exactly three labeled fermions, $\psi_{(\mu)}$, $\psi_{(\nu)}$, $\psi_{(\sigma)}$, in one of three labeled states 1, 2, 3.

Now, we draw a Feynman diagram from (10), using the following rules:

1) For the term $\frac{1}{q^2_{(\mu)}q^2_{(\nu)}}\left[\left(\overline{\psi}_{(\mu 2)}\lambda_j\gamma^\mu\psi_{(\mu 1)}\right)\left(\overline{\psi}_{(\nu 3)}\lambda_k\gamma^\nu\psi_{(\nu 2)}\right), q^\sigma\right]$ in the first line of the above, we draw this as an interaction between the current $\overline{\psi}_{(\mu 2)}\lambda_j\gamma^\mu\psi_{(\mu 1)}$ and the current $\overline{\psi}_{(\nu 3)}\lambda_k\gamma^\nu\psi_{(\nu 2)}$ mediated by the vector boson with momentum q^σ. We do the same for the other terms,

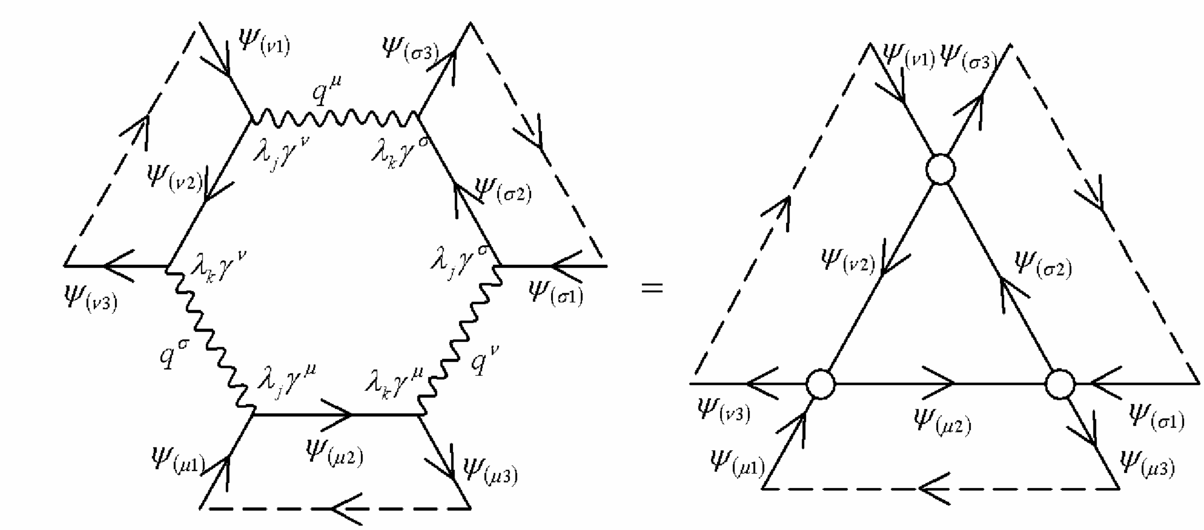
interconnecting all lines from all three terms. Following this rule, we find that q^{μ} ends up on the opposite side of the diagram from $\psi_{(\mu)}$, and similarly for q^{ν} from $\psi_{(\nu)}$ and q^{σ} from $\psi_{(\sigma)}$.

2) We place $\lambda_j \gamma^{\mu}$ at the vertex between $\psi_{(\mu 1)}$ and $\psi_{(\mu 2)}$, and $\lambda_k \gamma^{\nu}$ at the vertex between $\psi_{(\nu 2)}$ and $\psi_{(\nu 3)}$. We do the same for all other terms.

3) We draw a dashed line from state "3" to state "1", to represent an iterative "recycling" from state 3 back to state 1. One may think of this a being in the nature of a "finite state machine" which iteratively cycles from states 1 → 2 → 3/1 → 2 → 3/1 → 2 → 3.

4) We take this resulting diagram, remove the vector boson lines, and draw a, equivalent second diagram showing only the fermions in the form of interconnected, four-branch Mandelstam diagrams, so that all of the s, t, u channels are implicitly represented.

The resulting diagrams are below:



We find that a non-Abelian magnetic source $P_i^{\mu\nu\sigma}$ inherently contains exactly three fermion constituents, and, that this number of fermion constituents is *independent* of the rank of the non-Abelian gauge group. One ends up with $P_i^{\mu\nu\sigma}$ containing three fermions for any group SU(N), N>1. If we want each of these three fermion to possess a distinct quantum number for the purpose of satisfying Dirac statistics, then because there are three fermions in this magnetic source, we choose the color group SU(3). Regarding each of the fermions in the above diagram as being one "color" in this group, and the vector bosons as being massless, one must give serious consideration to the prospect that the above diagrams may, in reality, represent a baryon which naturally flows from the non-Abelian magnetic source $P_i^{\mu\nu\sigma}$.

If one adopts the "finite state machine" view in rule 3 above, then we may think of this "baryon candidate" as starting with each fermion (quark) in state 1. Because there are no gluons "in transit" in state 1, the total energy of the baryon consists entirely of the sum of the three quarks masses and their kinetic energies. Then, one can envision a gluon emitted from each quark in the 1→2 transition. Between emission and absorption (i.e., while gluons are in transit), the total energy of the baryon now consists of the sum of the three quarks masses, plus their kinetic energies, plus the potential energies between quarks as represented by the gluon energies. If one considers the higher order gluon / gluon interactions which are a distinctive feature of non-abelian gauge theories, then the self-coupling of the gluons contributes non-linearly to the potential energy. Once all the gluons are absorbed, the quarks are in state 2, and the energy of the baryon again consists entirely of the sum of the three quarks masses and their kinetic energies. This continues iteratively, millions of times per second. Because the whole baryon, viewed in isolation from any external influence, must conserve energy, the total baryon rest energy must remain unchanged throughout. This total rest energy, for the neutron and proton, for example, must remain fixed at 939.565 MeV and 938.273 MeV, respectively.

Next, one may specify a "duality" relationship between the third rank antisymmetric tensor $P_i^{\mu\nu\sigma}$, and a first-rank four vector:

$$*P_{i\tau} = \tfrac{1}{3!}\varepsilon_{\tau\mu\nu\sigma}P_i^{\mu\nu\sigma} \quad (11)$$

One can show that these first and third rank objects are self-dual, $*P_{i\tau} = P_{i\tau}$ (we shall not do so here, but look at section 4 in http://arxiv.org/abs/hep-ph/0508257 by the same author). $P_{i\tau}$ is in the nature of a non-Abelian four-vector current density, but this current is a magnetic rather than an electric current. Equation (11) also highlights the fact that even though $P_i^{\mu\nu\sigma}$ has 64 spacetime components (and 64 times *i* total components), only four of the spacetime components are independent. Combining equations (10) and (11), using $*P_{i\tau} = P_{i\tau}$, and writing $P_{i\tau}$ as a wavefunction equation $P_{i\tau} = \overline{\Psi}\lambda_i\gamma^\mu\Psi$, we now write:

$$P_{i\tau} = \overline{\Psi}\lambda_i\gamma^\mu\Psi = -i\tfrac{1}{3!}\varepsilon_{\tau\mu\nu\sigma}gf_{ijk}\begin{pmatrix} +\dfrac{1}{q^2{}_{(\mu)}q^2{}_{(\nu)}}\left[\left(\overline{\psi}_{(\mu 2)}\lambda_j\gamma^\mu\psi_{(\mu 1)}\right)\left(\overline{\psi}_{(\nu 3)}\lambda_k\gamma^\nu\psi_{(\nu 2)}\right), q^\sigma\right] \\ +\dfrac{1}{q^2{}_{(\nu)}q^2{}_{(\sigma)}}\left[\left(\overline{\psi}_{(\nu 2)}\lambda_j\gamma^\nu\psi_{(\nu 1)}\right)\left(\overline{\psi}_{(\sigma 3)}\lambda_k\gamma^\sigma\psi_{(\sigma 2)}\right), q^\mu\right] \\ +\dfrac{1}{q^2{}_{(\sigma)}q^2{}_{(\mu)}}\left[\left(\overline{\psi}_{(\sigma 2)}\lambda_j\gamma^\sigma\psi_{(\sigma 1)}\right)\left(\overline{\psi}_{(\mu 3)}\lambda_k\gamma^\mu\psi_{(\mu 2)}\right), q^\nu\right] \end{pmatrix}. \quad (12)$$

$P_{i\tau}$, of course, represents a true "monopole" in the sense of it being a "point" source at some suitable scale of observation, and, going back to (2), it is a magnetic monopole in the sense that it is derived directly from Maxwell's magnetic equation in non-Abelian form, written as:

$$P_{i\tau} = \tfrac{1}{3!}\varepsilon_{\tau\mu\nu\sigma}P_i^{\mu\nu\sigma} = \tfrac{1}{3!}\varepsilon_{\tau\mu\nu\sigma}\left(\partial^\sigma F_i^{\mu\nu} + \partial^\mu F_i^{\nu\sigma} + \partial^\nu F_i^{\sigma\mu}\right) \quad (13)$$

This leads one to consider that $P_{i\tau}$ may represent a baryon current density four-vector when viewed from a low-energy scale of observation wherein the individual baryons are effectively experienced as "point" sources, that the term on the right side of (12) represents an individual baryon when probed more closely such that its three fermion constituents and its "tripole" nature become apparent, that Ψ is perhaps the wavefunction for the entire baryon when that baryon is experienced as a point source, that the "monopole" and "tripole" views of a baryon are related by first / third rank duality, and that when we do observe a baryon from afar as a "point" source, we are effectively observing a non-Abelian magnetic monopole which is revealed on closer inspection to be a tripole object containing three fermion constituents.

Viewed in this way, it may well be that James Clerk Maxwell, unbeknownst to him (or, to this day, anyone else), through his magnetic equations, was the original discoverer of baryons. The theoretical tools which were lacking in his day to make this connection, however, were generally-covariant notation in spacetime, and non-Abelian gauge theory. The experimental data which was lacking, was that the proton and the neutron has not yet been observed, and that these nucleons consist of three confined fermions (partons). With these connections, Maxwell's equations may well be the foundation not only for the physics of electricity, but also, the foundation for nuclear physics.

Finally, the above may yield an avenue to simultaneously consider why QCD is a short range interaction and also, why quarks are confined. Consider that for weak interactions, the masses M_W and M_Z of the massive W and Z vector bosons sits in the denominator of the propagator and hence in the denominator of the invariant amplitude. This establishes the short range of the weak interaction. In equation (12), multiplier terms such as $\frac{1}{q^2{}_{(\mu)} q^2{}_{(\nu)}}$ play a role analogous to the propagator for bosons interacting with free fermions. Because the gluons are considered to be massless, i.e., $q^2 = 0$, these multiplier terms are formally equal to infinity. This leads to the same set of issues confronted in quantum field theory wherein one uses the prescription of an imaginary "lifetime" term $i\varepsilon$ to steer around these "poles." Alternatively, Dirac delta functions $\delta(q^\sigma)$ which are functions of q^σ are employed to deal with infinities which arise from denominator terms of the form $q^2 = 0$, and can be made into "pulses" which are finite in both width and amplitude yet very narrow. If one of these known approaches can be applied to render the terms $\frac{1}{q^2{}_{(\mu)} q^2{}_{(\nu)}}$ *finite*, then these terms will now simultaneously determine the range of the interactions between the quarks which comprise a baryon, and will simultaneously maintain the quarks in a confined state within that same range. Were this avenue to be capable of establishing a range on the order of 1 fermi, one would then have arrived at the foundations of both quark confinement and the short range of the nuclear force despite the masslessness of the gluons.