Paper Preview: Five Dimensional Spacetime with Axial Time, and the Geometric Origin of Mass

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Here is a very quick summary of a paper I am planning regarding the Dirac "axial" operator γ^5 as a fifth spacetime dimension that gives a geometric interpretation to mass.

Spacetime metric is (U,V = 0, 1, 2, 3, 5):

$$dT^2 \equiv g_{UV}dx^U dx^V = dx^U dx_U;\ g_{UV} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

with signature determined by the Dirac gamma matrices:

$$½[\gamma^U\gamma^V + \gamma^V\gamma^U] \equiv g^{UV}. \quad (2)$$

In essence, we have made the hypothesis that γ^5 *is of such fundamental origin, that it implies nothing less than a requirement to add a fifth dimension to spacetime*, and to define the metric invariants using four spacetime dimensions plus one axial dimension rather than merely four spacetime dimensions. Based on the metric signature, this fifth axial dimension is timelike.

With the usual spacetime metric $d\tau^2 \equiv g_{uv}dx^u dx^v$, eq. (1) becomes:

$$dT^2 = d\tau^2 + dx^5 dx_5;\ \text{i.e.,}\ \frac{d\tau^2}{dT^2} = 1 - \frac{dx^5 dx_5}{dT^2},\ \text{i.e.,}\ \frac{d\tau}{dT} = \sqrt{1 - \frac{dx^5 dx_5}{dT^2}}\ . \quad (3)$$

If we use the four-velocity $u^\mu \equiv dx^\mu/d\tau$, then if we divide through by dT^2, equation (1) also becomes:

$$1 = \frac{dx^\mu dx_\mu}{dT^2} + \frac{dx^5 dx_5}{dT^2} = \frac{dx^\mu dx_\mu}{d\tau^2}\frac{d\tau^2}{dT^2} + \frac{dx^5 dx_5}{dT^2} = u^\mu u_\mu \frac{d\tau^2}{dT^2} + \frac{dx^5 dx_5}{dT^2} \quad (4)$$

Isolating 0 on the right, and using (3), we then get:

$$0 = u^\mu u_\mu \frac{d\tau^2}{dT^2} - \left(1 - \frac{dx^5 dx_5}{dT^2}\right) = u^\mu u_\mu \frac{d\tau^2}{dT^2} - \frac{d\tau^2}{dT^2} = \left(u^\mu u_\mu - 1\right)\frac{d\tau^2}{dT^2} \quad (5)$$

All traces of the fifth dimension have dropped out of this equation in which the indexes are summed over the usual four spacetime dimensions only. The fifth dimension is contained implicitly in the scalar ratio $\frac{d\tau^2}{dT^2}$, *which Lorentz transforms in the same way as a mass!* So, in some way, we are hitting on the geometric origin of mass, and need to understand this better.

Normally, we define the momentum four-vector $P^\mu \equiv mu^\mu$, and the usual counterpart to equation (5) is:

$$0 = \left(u^\mu u_\mu - 1\right)m^2 = P^\mu P_\mu - m^2 \tag{6}$$

It is the contrast of (5) and (6) which is crucial. Equation (6) has a mass in it. That mass enters "by hand." Later, the m^2 in equation (6) becomes the μ^2 in the Klein Gordon equation that is set to $\mu^2<0$ for symmetry breaking. In the Dirac equation, m becomes the Fermion rest mass that we have to remove and replace with, e.g., Halzen and Martin's equation 15.31) or (15.35). In short, setting $P^\mu \equiv mu^\mu$ in the Einstein metric to get to equation (6) is the "original sin" that later leads us to have to do all sorts of funny thing to reveal a mass rather than have it put in by hand from the start. Equation (6) is fine if you already have a mass that you just want to plug in and see what the dynamics are. But, if you want to *deduce* a mass, then using $P^\mu \equiv mu^\mu$ to lead to (6) is a fateful step that we then have to work around in QFT.

Equation (6), of course, becomes an operator Klein-Gordon equation in QFT and its "square root" $\left(\gamma^\mu P_\mu - m\right)\psi = \left(i\gamma^\mu \partial_\mu - m\right)\psi = 0$ used as an operator becomes Dirac's equation in QFT. So, the whole machinery of the equations used to derive QFT solutions originates in equation (6).

In equation (5), we see that $\frac{d\tau^2}{dT^2}$ Lorentz transforms *precisely* in the same manner as a mass, and yet it is a totally geometric quantity in 5-dimensions. That is, if we cryptically define:

$$"m^2" \equiv \frac{d\tau^2}{dT^2}; \quad "m" \equiv \frac{d\tau}{dT}; \quad "P^\mu" \equiv \frac{d\tau}{dT}u^\mu; \quad "\psi" \equiv ue^{-i"P^\mu"x_\mu}, \tag{7}$$

then we can have a totally geometric "mass precursor" $"m" \equiv \frac{d\tau}{dT}$ which Lorentz transforms just like a mass show up everywhere, and it will not have been introduced by hand. In these terms, Dirac's equation is:

$$\left(\gamma^\mu "P_\mu" - "m"\right)"\psi" = \left(i\gamma^\mu \partial_\mu - "m"\right)"\psi" = 0, \tag{8}$$

and everything else (gauge symmetry, symmetry breaking etc.) starts from there.

This, of course, does not tell us how to generate mass, and that is still a difficult problem to be worked through. But, it points out the fundamental origin of the problem of masses input by hand, the moment we set $P^\mu \equiv mu^\mu$. By using $"m^2" \equiv \frac{d\tau^2}{dT^2}$ to denote the mass-like transformation properties of $\frac{d\tau^2}{dT^2}$, we can now try to understand mass as motion of a particle in the 5-dimensional manifold, $\Re^5$.

Feynman first came up with the idea that a particle can move forward or backward in time. Here, dx^5 is a *second* timelike dimension, and time therefore is a plane, and particles move at various angles through this "time plane." "Forward" and "backward" are merely subsets of this. A photon or any massless particle, $d\tau=0$, moves entirely through the axial dimension, that is, via (3), $0 = 1 - \frac{dx^5 dx_5}{dT^2}$, i.e., $dT=dx^5$. Particles with non-zero mass move partly through dt and partly through dx^5, with more movement through dt and less through dx^5 as the mass increases. (In fact, one can develop mathematics for rotation of dT through the dt and dx^5 time plane through a simple angle θ.)

So, that is the basic idea. This goes straight to the heart of the structure of spacetime, because it pinpoints the "original sin" which leads to problems with generating mass later on, and so may give us some new insights to work through the mass problems, because it gives us a completely geometric way to think about mass. We know that gamma-5 does all sorts of things, including project out two-component massless Dirac spinors. So, the fact that gamma-5 shows up at this level and also relates to mass may contain many new and important clues.

In contrast to other 5-or-more dimensional theories including Weyl's original work where the fifth dimension is "curled up,", the fifth dimension here derives directly from the γ^5 matrices where we regard the Dirac γ^U, U=0,1,2,3,5, via equation (2), as the structure generators of a five-dimensional spacetime with axial time. This fifth, axial time dimension, together with the ordinary time dimension, defines a "time plane," and the mass of a particle is understood to bear a relationship to how that particle moves through the time plane, relatively to how we, as observers, move through the time plane.